Eur. Phys. J. C 53, 95–97 (2008)
DOI 10.1140/epjc/s10052-007-0435-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Two dimensional black hole entropy

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Received: 2 September 2007 /
Published online: 24 October 2007 –

Abstract. In this paper we consider the two dimensional black hole and use Wald's formula to find its entropy. Also we consider higher derivative terms. We discuss the case of a massless black hole and find its free energy.

1 Introduction

For the two dimensional black hole the horizon is a point, so the horizon area simply vanishes and we cannot use the ordinary method to find the black hole entropy, because it is proportional to the area at the horizon; hence its entropy seems to be zero. Although the possibility of describing 2D black holes by means of a CFT has been widely investigated [1–7], it is not completely clear if it is always possible to mimic the gravitational dynamics of the 2D black hole through a CFT. However, in two dimensional dilaton gravity theories [8–10], it has been shown that the entropy is proportional to the value of the dilaton field at the horizon. Recently one found that the entropy function [11] can be derived from Wald's formula [12], and to this aim one first rewrites the Lagrangian density in terms of values of fields near the horizon, then taking the Legendre transform of the resulting function with respect to the electric field with multiplication by an overall factor of 2π. The near horizon geometry of the extremal black hole is determined by extremizing the entropy function, and the black hole entropy is given by the extremum value of the entropy function. This general method is an easier way to calculate the black hole entropy. In the next section we use this method for two dimensional gravity in AdS space and find the entropy.

2 Two dimensional black hole

First we briefly review the approach of the entropy function. Entropy function analysis provides a good understanding of the attractor mechanism for spherically symmetric extremal black holes if we have the following.

1. We consider a theory of gravity coupled to abelian (p-form) gauge fields and neutral scalar fields.
2. The Lagrangian density f is gauge and general coordinate invariant.
3. Define an extremal black hole to be one whose near horizon geometry is $\mathrm{AdS}_2 \times \mathrm{S}^2$ (in $D = 4$).

In this approach, the theory need not be supersymmetric, and f could contain higher derivative terms. For such black holes one can define an entropy function E as follows:

$$E = 2\pi[q_i \epsilon_i - f] , \tag{1}$$

where the q_i denote the electric charges, and the ϵ_i are the near horizon radial electric fields. E is a function of the q_i and various parameters labeling the $\mathrm{SO}(2,1) \times \mathrm{SO}(3)$ symmetric near horizon background (e.g. the sizes of AdS_2 and S^2, and the vacuum expectation values of scalars, radial electric fields, and radial magnetic fields). Then for a black hole with given electric charges q and magnetic charges p, all other near horizon parameters are obtained by extremizing E with respect to these parameters. Finally the entropy is given by the value of E at its extremum.

In two dimensions the Einstein action is a topological invariant, and hence it has no dynamical content. The simplest way to obtain the dynamics is to include a dilaton field. An example of such a theory naturally arises within the framework of the usual four dimensional Einsteinian gravity if we restrict ourselves to the consideration of spherically symmetric spacetimes. Here we consider two dimensional gravity with negative cosmological constant ($\Lambda = -\frac{3}{l^2}$) and a scalar field coupled to a gauge field described by the action

$$I = \int \mathrm{d}^2x \sqrt{-g} \times \left[\frac{R + 6l^{-2}}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} U(\phi) F^2 \right] , \tag{2}$$

where l is the AdS radius, and G is Newton's constant. The near horizon solution of the extremal black hole with

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charge q can generically be written in the form

$$\begin{aligned} \mathrm{d}s^2 &= v\left(-g(r)\,\mathrm{d}t^2 + \frac{1}{g(r)}\,\mathrm{d}r^2\right), \\ \phi &= u, \\ F_{rt} &= \epsilon, \\ g(r) &= \frac{r^2}{l^2} - 1\,, \end{aligned} \tag{3}$$

where u, v and ϵ are constants to be determined in terms of the charge q. Now we will use Wald's formula to determine the entropy function. Therefore, first by using (3) we write the Lagrangian density in terms of u, v and ϵ,

$$f(u,v,\epsilon) = v\left[\frac{(-\frac{2}{v}+6)l^{-2}}{16\pi G} - V(u) + \frac{U(u)}{2v^2}\epsilon^2\right]. \tag{4}$$

Next, the entropy function [11] is defined as the Legendre transform of the Lagrangian density with respect to the gauge field ϵ,

$$\begin{aligned} E(u,v,\epsilon) &= 2\pi[q\epsilon - f(u,v,\epsilon)] \\ &= v\left[\frac{\pi q^2}{U(u)} - \frac{(-\frac{2}{v}+6)l^{-2}}{8G} + 2\pi V(u)\right], \end{aligned} \tag{5}$$

where $q = \frac{\partial f}{\partial \epsilon} = \frac{\epsilon U(u)}{v}$ denotes the electric charge carried by the black hole. The undetermined parameters u and v can be fixed by the equation of motion, which becomes the extremum equation as follows:

$$\begin{aligned} \frac{\partial E}{\partial v}(u_e,v_e) &= \left[\frac{\pi q^2}{U(u_e)} - \frac{(-\frac{2}{v_e}+6)l^{-2}}{8G} + 2\pi V(u_e)\right] - \frac{2l^{-2}}{8Gv_e} \\ &= 0\,, \end{aligned} \tag{6}$$

where u_e and v_e are the extrema of u and v, respectively. Therefore we have

$$\left[\frac{\pi q^2}{U(u_e)} - \frac{(-\frac{2}{v_e}+6)l^{-2}}{8G} + 2\pi V(u_e)\right] = \frac{2l^{-2}}{8Gv_e}. \tag{7}$$

The entropy is given by the value of the entropy function of the extremum,

$$S_{\mathrm{BH}} = E(u_e,v_e) = \frac{2}{8Gl^2}, \tag{8}$$

which is obtained by inserting (7) in (5). Note that the entropy is proportional to $\frac{1}{G}$ generally, and the effective two dimensional Newton constant is given by $G = \frac{2\pi}{\langle\phi\rangle}$ [13], where $\langle\phi\rangle$ is the expectation value of the dilaton field. The value of the dilaton field at the horizon is u_e. Therefore, the entropy is proportional to u_e, as is expected.

3 Higher derivative terms

Now let us consider the effect of the higher derivative terms. For this goal it is sufficient to consider the higher derivative terms of the form R^n. This means that we must do the replacement $R \to \sum a_n R^n = R + a_2 R^2 + \cdots$. Hence the change of action due to higher derivative terms is

$$\Delta I = \int \mathrm{d}^2x\sqrt{-g}\left[\frac{\sum a_n R^n}{16\pi G}\right], \tag{9}$$

and

$$\Delta f = v\left[\frac{\sum a_n(\frac{2l^{-2}}{v})^n}{16\pi G}\right]. \tag{10}$$

Then the entropy (8) is modified to

$$S_{\mathrm{mod}} = \frac{1}{8G}\left[2l^{-2} - \sum n a_n(-2l^{-2})^n v_e^{1-n}\right]. \tag{11}$$

Now we will give an interesting example. A static black hole solution with topology $R^2 \times \Sigma$, where Σ is a two dimensional manifold of negative constant curvature, is given by

$$\begin{aligned} \mathrm{d}s^2 = \frac{r(r+2G\mu)}{(r+G\mu)^2}\Bigg[&-\left(\frac{r^2}{l^2} - \left(1+\frac{G\mu}{r}\right)^2\right)\mathrm{d}t^2 \\ &+ \left(\frac{r^2}{l^2} - \left(1+\frac{G\mu}{r}\right)^2\right)^{-1}\mathrm{d}r^2\Bigg], \end{aligned} \tag{12}$$

where the constant μ stands for the mass of the black hole. If we compare the metric (12) with (3) we find

$$\begin{aligned} v &= \frac{r(r+2G\mu)}{(r+G\mu)^2}, \\ g(r) &= \frac{r^2}{l^2} - \left(1+\frac{G\mu}{r}\right)^2. \end{aligned} \tag{13}$$

We see that the value of v is constant only for $\mu = 0$. The configuration (12) is the one of asymptotically locally AdS spacetimes. For $\mu = 0$, these spacetimes admit Killing spinors provided Σ is a noncompact surface [14], and such a configuration describes the supersymmetric ground state of a wrapped black string and is therefore expected to be stable. As was shown in [15] these configurations are also stable under gravitational perturbations, for $\mu \neq 0$ AdS spacetimes, with a single timelike Killing vector ∂_t, provided Σ is assumed to be compact without boundary. When $\mu = 0$, the event horizon is given by $r_+ = l$ and the temperature,

$$T = \frac{1}{2\pi l}\left(\frac{2r_+}{l} - 1\right), \tag{14}$$

which reduces to $T = \frac{1}{2\pi l}$. Then from (13) we see that $v = 1$ and $g(r) = \frac{r^2}{l^2} - 1$; therefore the modified entropy becomes

$$S_{\mathrm{mod}} = \frac{1}{8G}\left[\frac{2}{l^2} - \sum n a_n\left(-\frac{2}{l^2}\right)^n\right]. \tag{15}$$

We will find the free energy F in this limit. For a massless black hole we know that $I = S - \beta\mu = -\beta F$, where $\beta = \frac{1}{T}$ ($\hbar = k_{\mathrm{B}} = 1$). Hence for a massless black hole we obtain

$$F = -\frac{1}{16\pi Gl}\left[\frac{2}{l^2} - \sum n a_n \left(-\frac{2}{l^2}\right)^n\right]. \quad (16)$$

This is the free energy of a two dimensional static and massless black hole obtained by Wald's method.

4 Conclusion

In this paper we have used Sen's entropy function to obtain the entropy of a two dimensional charged extremal black hole. We have shown that the entropy is proportional to the value of the dilaton field at the horizon. Next we considered the effect of higher derivative terms and found the modified entropy. Then we applied this method to a static black hole and saw that these black hole must be massless; after that we obtained the free energy of a black hole by using the modified entropy.

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